# Why is TeV-scale a geometric mean of neutrino mass and GUT-scale?

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#### Abstract

Among three typical energy scales, a neutrino mass scale  $(m_{\nu} \sim 0.1 \text{ eV})$ , a GUT scale  $(M_{GUT} \sim 10^{16} \text{ GeV})$ , and a TeV-scale  $(M_{NP} \sim 1 \text{ TeV})$ , there is a fascinating relation of  $M_{NP} \simeq \sqrt{m_{\nu} \cdot M_{GUT}}$ . The TeV-scale,  $M_{NP}$ , is a new physics scale beyond the standard model which is regarded as supersymmetry in this letter. We suggest a simple supersymmetric neutrinophilic Higgs doublet model, which realizes the above relation dynamically as well as the suitable  $m_{\nu}$  through a tiny vacuum expectation value of neutrinophilic Higgs without additional scales other than  $M_{NP}$  and  $M_{GUT}$ . A gauge coupling unification, which is an excellent feature in the supersymmetric standard model, is preserved automatically in this setup.

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#### 1 Introduction

There are three typical energy scales, a neutrino mass scale ( $m_{\nu} \sim 0.1 \text{ eV}$ ), a GUT scale ( $M_{GUT} \sim 10^{16} \text{ GeV}$ ), and a TeV-scale ( $M_{NP} \sim 1 \text{ TeV}$ ) which is a new physics scale beyond the standard model (SM) and regarded as supersymmetry (SUSY) in this letter. Among these three scales, we notice a fascinating relation,

$$M_{NP}^2 \simeq m_{\nu} \cdot M_{GUT} \ . \tag{1}$$

Is this relation an accident, or providing a clue to the underlying new physics? We take a positive stance toward the latter possibility.

As for a neutrino mass  $m_{\nu}$ , its smallness is still a mystery, and it is one of the most important clues to find new physics. Among a lot of possibilities, a neutrinophilic Higgs doublet model suggests an interesting explanation of the smallness by a tiny vacuum expectation value (VEV) [1]-[15]. This VEV from a neutrinophilic Higgs doublet is of  $\mathcal{O}(0.1)$  eV which is the same as the neutrino mass, so that it suggests Dirac neutrino[3, 4, 6, 8].\* Thus, the neutrino mass is much smaller than other fermions, since its origin is the tiny VEV from the different (neutrinophilic) Higgs doublet. Introduction of  $Z_2$ -symmetry distinguishes the neutrinophilic Higgs from the SM-like Higgs, where  $m_{\nu}$  is surely generated only through the VEV of the neutrinophilic Higgs. The SUSY extension of the neutrinophilic doublet model is considered in Refs.[7, 11, 12, 15]. Since the neutrino Yukawa couplings are not necessarily tiny anymore, some related researches have been done, such as, collider phenomenology[8, 10], low energy thermal leptogenesis[11, 12], cosmological constraints[13]<sup>†</sup>, and so on.

On the other hand, for the SUSY, it is the most promising candidate as a new physics beyond the SM because of a excellent success of gauge coupling unification. Thus, the SUSY SM well fits the GUT scenario as well as an existence of a dark matter candidate.

There are some attempts that try to realize the relation in Eq.(1). One example is to derive  $m_{\nu}$  from a higher dimensional operator in the SUSY framework[16]. Another example is to take a setup of matter localization[17] in a warped extra dimension[18]. Both scenarios are interesting, but a model in this letter is much simpler and contains no additional scales other than  $M_{NP}$ ,  $m_{\nu}$ , and  $M_{GUT}$ . (For other related papers, see, for example, [19, 20].)

In this letter, we suggest a simple SUSY neutrinophilic Higgs doublet model, which dynamically realizes the relation of Eq.(1). Usually, SUSY neutrinophilic doublet models have tiny mass scale of soft  $Z_2$ -symmetry breaking ( $\rho, \rho' = \mathcal{O}(10)$  eV in Refs.[11, 12, 15]). This additional tiny mass scale plays a crucial role of generating the tiny neutrino mass, however, its origin is completely unknown (and assumption). In other words, the smallness of  $m_{\nu}$  is

<sup>\*</sup> In Refs.[1, 2, 5, 9, 10, 11, 12], Majorana neutrino scenario is considered through TeV-scale seesaw with a neutrinophilic Higgs VEV of  $\mathcal{O}(1)$  MeV.

<sup>&</sup>lt;sup>†</sup> A setup in Refs.[13] is different from usual neutrinophilic Higgs doublet models, since it includes a light Higgs particle.

just replaced by that of  $Z_2$ -symmetry breaking mass parameters, and this is not an essential explanation of tiny  $m_{\nu}$ . This is a common serious problem exists in neutrinophilic Higgs doublet models in general. Our model solves this problem, where two scales of  $M_{GUT}$  and  $M_{NP}$  naturally induce the suitable magnitude of  $m_{\nu}$  through the relation of Eq.(1), and does not require any additional scales. The model contains a pair of new neutrinophilic Higgs doublets with GUT-scale masses, and the  $Z_2$ -symmetry is broken by TeV-scale dimensionful couplings of these new doublets to the ordinary SUSY Higgs doublets. Once the ordinary Higgs doublets obtain VEVs ( $v_{u,d}$ ) by the usual electroweak symmetry breaking, it triggers VEVs for the neutrinophilic Higgs doublets of size,  $m_{\nu} \sim v_{u,d} M_{NP}/M_{GUT}$ . Then,  $\mathcal{O}(1)$  Yukawa couplings of the neutrinophilic doublets to L N (L: lepton doublet, N: right-handed neutrino) give neutrino masses of the proper size. A gauge coupling unification is also preserved automatically in our setup.

# 2 SUSY neutrinophilic Higgs doublet model

At first, we show a SUSY neutrinophilic Higgs doublet model in a parameter region which is different from Refs.[11, 12, 15]. We introduce  $Z_2$ -parity, where only vector-like neutrinophilic Higgs doublets and right-handed neutrino have odd-charge. The superpotential of the Higgs sector is given by<sup>‡</sup>

$$W_h = \mu H_u H_d + M H_{\nu} H_{\nu'} - \rho H_u H_{\nu'} - \rho' H_{\nu} H_d.$$
 (2)

 $H_{\nu}$  ( $H_{\nu'}$ ) is a neutrinophilic Higgs doublet, and  $H_{\nu}$  has Yukawa interaction of  $LH_{\nu}N$ , which induces a tiny Dirac neutrino mass through the tiny VEV,  $\langle H_{\nu} \rangle$ . This is the origin of smallness of neutrino mass, and this paper devotes a Dirac neutrino scenario, i.e.,  $m_{\nu} \simeq \langle H_{\nu} \rangle = \mathcal{O}(0.1)$  eV. On the other hand,  $H_{\nu'}$  does not couple with any matters.  $H_{u}$  and  $H_{d}$  are Higgs doublets in the minimal SUSY SM (MSSM), and quarks and charged lepton obtain their masses through  $\langle H_{u} \rangle$  and  $\langle H_{d} \rangle$ . Note that this structure is guaranteed by the  $Z_{2}$ -symmetry. Differently from conventional neutrinophilic Higgs doublet models, we here take M the GUT scale and  $\mu, \rho, \rho'$   $\mathcal{O}(1)$  TeV. The soft  $Z_{2}$ -parity breaking parameters,  $\rho$  and  $\rho'$ , might be induced from SUSY breaking effects (which will be discussed in the next section), and we regard  $\rho$  and  $\rho'$  as mass parameters of new physics scale,  $M_{NP} = \mathcal{O}(1)$  TeV. Usually, SUSY neutrinophilic doublet models take  $\rho, \rho' = \mathcal{O}(10)$  eV (for  $\mathcal{O}(1)$  TeV B-terms)[11, 12, 15]. This additional tiny mass scale plays a crucial role of generating the tiny neutrino mass however, its origin is just an assumption. Thus, the smallness of  $m_{\nu}$  is just replaced by that of  $\rho, \rho'$ . This is a common serious problem exists in neutrinophilic Higgs doublet models in general. The present model solves this problem, in which two scales of  $M_{GUT}$  and  $M_{NP}$  induce the suitable magnitude of

<sup>&</sup>lt;sup>‡</sup> The author would like to thank R. Kitano for pointing out a paper[21], which suggested the similar model and also estimated lepton flavor violating processes.

 $m_{\nu}$  dynamically, and does not require any additional scales, such as  $\mathcal{O}(10)$  eV. This is one of the excellent points in our model.

The potential of the Higgs doublets is given by

$$V = (|\mu|^{2} + |\rho|^{2})H_{u}^{\dagger}H_{u} + (|\mu|^{2} + |\rho'|^{2})H_{d}^{\dagger}H_{d} + (|M|^{2} + |\rho'|^{2})H_{\nu}^{\dagger}H_{\nu} + (|M|^{2} + |\rho|^{2})H_{\nu'}^{\dagger}H_{\nu'}$$

$$+ \frac{g_{1}^{2}}{2} \left( H_{u}^{\dagger} \frac{1}{2} H_{u} - H_{d}^{\dagger} \frac{1}{2} H_{d} + H_{\nu}^{\dagger} \frac{1}{2} H_{\nu} - H_{\nu'}^{\dagger} \frac{1}{2} H_{\nu'} \right)^{2}$$

$$+ \sum_{a} \frac{g_{2}^{2}}{2} \left( H_{u}^{\dagger} \frac{\tau^{a}}{2} H_{u} + H_{d}^{\dagger} \frac{\tau^{a}}{2} H_{d} + H_{\nu}^{\dagger} \frac{\tau^{a}}{2} H_{\nu} + H_{\nu'}^{\dagger} \frac{\tau^{a}}{2} H_{\nu'} \right)^{2}$$

$$- m_{H_{u}}^{2} H_{u}^{\dagger} H_{u} + m_{H_{d}}^{2} H_{d}^{\dagger} H_{d} + m_{H_{\nu}}^{2} H_{\nu}^{\dagger} H_{\nu} + m_{H_{\nu'}}^{2} H_{\nu'}^{\dagger} H_{\nu'}$$

$$+ B\mu H_{u} \cdot H_{d} + B' M H_{\nu} \cdot H_{\nu'} - \hat{B} \rho H_{u} \cdot H_{\nu'} - \hat{B}' \rho' H_{\nu} \cdot H_{d}$$

$$- \mu^{*} \rho H_{d}^{\dagger} H_{\nu'} - \mu^{*} \rho' H_{u}^{\dagger} H_{\nu} - M^{*} \rho' H_{\nu'}^{\dagger} H_{d} - M^{*} \rho H_{\nu}^{\dagger} H_{u} + \text{h.c.}, \tag{3}$$

where  $\tau^a$  and dot mean a generator and cross product of SU(2), respectively.  $m_{H_u}^2$ ,  $m_{H_d}^2$ ,  $m_{H_\nu}^2$ ,  $m_{H_\nu}^2$ , B, B',  $\hat{B}$ , and  $\hat{B}'$  are soft SUSY breaking parameters of order  $\mathcal{O}(1)$  TeV. We assume  $(-m_{H_u}^2) < 0$  for the suitable electroweak symmetry breaking, and real VEVs as  $\langle H_u \rangle = v_u$ ,  $\langle H_d \rangle = v_d$ ,  $\langle H_{\nu} \rangle = v_{\nu}$ ,  $\langle H_{\nu'} \rangle = v_{\nu'}$  in neutral components. Now let us examine whether we can really obtain the suitable magnitudes of VEVs as  $v_{u,d} = \mathcal{O}(10^2)$  GeV and  $v_{\nu,\nu'} = \mathcal{O}(0.1)$  eV or not. By taking  $\mu$ -,  $\rho$ -, B-parameters to be real, and denoting  $M_u^2 \equiv \mu^2 + \rho^2 - m_{H_u}^2 (< 0)$ ,  $M_d^2 \equiv \mu^2 + \rho'^2 + m_{H_d}^2 (> 0)$ ,  $M_{\nu}^2 \equiv M^2 + \rho'^2 - m_{H_u}^2 \simeq M^2 (> 0)$ , and  $M_{\nu'}^2 \equiv M^2 + \rho^2 + m_{H_d}^2 \simeq M^2 (> 0)$ , the stationary conditions of  $\frac{1}{2} \frac{\partial V}{\partial v_u} = 0$ ,  $\frac{1}{2} \frac{\partial V}{\partial v_d} = 0$ ,  $\frac{1}{2} \frac{\partial V}{\partial v_{\nu'}} = 0$  are given by

$$0 = M_u^2 v_u + \frac{1}{4} (g_1^2 + g_2^2) v_u (v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) + B\mu v_d - \hat{B}\rho v_{\nu'} - (\mu\rho' + M\rho) v_\nu, \tag{4}$$

$$0 = M_d^2 v_d - \frac{1}{4} (g_1^2 + g_2^2) v_d (v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) + B\mu v_u - \hat{B}' \rho' v_\nu - (\mu \rho + M \rho') v_{\nu'}, \tag{5}$$

$$0 = M_{\nu}^{2} v_{\nu} + \frac{1}{4} (g_{1}^{2} + g_{2}^{2}) v_{\nu} (v_{u}^{2} - v_{d}^{2} + v_{\nu}^{2} - v_{\nu'}^{2}) + B' M v_{\nu'} - \hat{B}' \rho' v_{d} - (\mu \rho' + M \rho) v_{u}, \quad (6)$$

$$0 = M_{\nu'}^2 v_{\nu'} - \frac{1}{4} (g_1^2 + g_2^2) v_{\nu'} (v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) + B' M v_\nu - \hat{B} \rho v_u - (\mu \rho + M \rho') v_d, \quad (7)$$

respectively. Regarding M is a GUT scale and  $v_{\nu}, v_{\nu'} \ll v_u, v_d$ , Eqs.(6) and (7) become

$$0 = Mv_{\nu} - \rho v_{u}, \quad 0 = Mv_{\nu'} - \rho' v_{d} \tag{8}$$

in the leading order, respectively. These are just the relation in Eq.(1)! (Here we neglect one order magnitude between  $M_{NP}$  and the weak scale.) This is what we want to derive, and the VEVs of neutrinophilic Higgs fields become

$$v_{\nu} = \frac{\rho v_u}{M}, \quad v_{\nu'} = \frac{\rho' v_d}{M}. \tag{9}$$

It is worth noting that they are induced dynamically through the stationary conditions in Eqs.(6) and (7), and their magnitudes are surely of  $\mathcal{O}(0.1)$  eV. As for  $v_{u,d}$ , by use of Eq.(9), Eqs.(4) and (5) become

$$0 = (M_u^2 - \rho^2)v_u + \frac{1}{4}(g_1^2 + g_2^2)v_u(v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) + B\mu v_d, \tag{10}$$

$$0 = (M_d^2 - \rho'^2)v_d - \frac{1}{4}(g_1^2 + g_2^2)v_d(v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) + B\mu v_u$$
(11)

in the leading order, respectively. Then, the MSSM Higgs fields take VEVs as

$$v^{2} \simeq \frac{2}{g_{1}^{2} + g_{2}^{2}} \left( \frac{M_{u}^{2} - M_{d}^{2}}{\cos 2\beta} - (M_{u}^{2} + M_{d}^{2}) \right), \quad \sin 2\beta \simeq \frac{2B\mu}{M_{u}^{2} + M_{d}^{2}}, \quad (12)$$

where  $v^2 = v_u^2 + v_d^2$ ,  $\tan \beta = v_u/v_d$ ,  $M_u'^2 \equiv M_u^2 - \rho^2$  and  $M_d'^2 \equiv M_d^2 - \rho'^2$ . They mean slight modifications of VEVs for  $H_u$  and  $H_d$ .

Since the masses of neutrinophilic Higgs  $H_{\nu}$  and  $H_{\nu'}$  are super-heavy as the GUT scale, there are no other vacua (such as,  $v_{u,d} \sim v_{\nu,\nu'}$ ) except for  $v_{u,d} \gg v_{\nu,\nu'}$  [15]. Also, their heaviness guarantees the stability of the VEV hierarchy,  $v_{u,d} \gg v_{\nu,\nu'}$ , against radiative corrections [14, 15]. It is because, in the effective potential,  $H_{\nu}$  and  $H_{\nu'}$  inside loop-diagrams are suppressed by their GUT scale masses. Anyhow, we stress again that the relation of Eq.(1) is dynamically obtained in Eq.(8).

As for the gauge coupling unification, it is preserved automatically in our setup, since fields except for the MSSM have masses of order the GUT scale.

# 3 SU(5) GUT embedded model

The model we suggested has the GUT scale mass of neutrinophilic Higgs doublets in Eq.(2), so that it is natural to embed the model in a GUT framework. Let us consider SU(5) GUT. A superpotential of a Higgs sector at the GUT scale is given by

$$W_H^{\text{GUT}} = M_0 \text{tr} \Sigma^2 + \lambda \text{tr} \Sigma^3 + H \Sigma \bar{H} + \Phi_{\nu} \Sigma \bar{\Phi}_{\nu} - M_1 H \bar{H} - M_2 \Phi_{\nu} \bar{\Phi}_{\nu}, \tag{13}$$

where  $\Sigma$  is an adjoint Higgs whose VEV reduces the GUT gauge symmetry into the SM.  $\Phi_{\nu}$  ( $\bar{\Phi}_{\nu}$ ) is a neutrinophilic Higgs of (anti-)fundamental representation, which contains  $H_{\nu}$  ( $H'_{\nu}$ ) in the doublet component (while the triplet component is denoted as  $T_{\nu}$  ( $\bar{T}_{\nu}$ )).  $\Phi_{\nu}$  and  $\bar{\Phi}_{\nu}$  are odd under the  $Z_2$ -parity. H ( $\bar{H}$ ) is a Higgs of (anti-)fundamental representation, which contains  $H_u$  ( $H_d$ ) in the doublet component (while the triplet component is denoted as T ( $\bar{T}$ )). The VEV of  $\Sigma$  and  $M_{0,1,2}$  are all of  $\mathcal{O}(10^{16})$  GeV, thus we encounter so-called triplet-doublet (TD) splitting problem. Some mechanisms have been suggested for a solution of TD splitting, but here we show a case that the TD splitting is realized just by a fine-tuning between  $\langle \Sigma \rangle$  and

 $M_1$ . That is,  $\langle \Sigma \rangle - M_1$  induces GUT scale masses of  $T, \bar{T}$ , while weak scale masses of  $H_u, H_d$ . This is a serious fine-tuning, so that we can not expect a simultaneous fine-tuned cancellation also happens between  $\langle \Sigma \rangle$  and  $M_2$ . Thus, we consider a case that the TD splitting only works in H and  $\bar{H}$ , while not works in  $\Phi_{\nu}$  and  $\bar{\Phi}_{\nu}$ . This situation makes Eq.(13) become

$$W_H^{eff} = \mu H_u H_d + M H_{\nu} H_{\nu'} + M' T \bar{T} + M'' T_{\nu} \bar{T}_{\nu}. \tag{14}$$

This is the effective superpotential of the Higgs sector below the GUT scale, and M, M', M'' are of  $\mathcal{O}(10^{16})$  GeV, while  $\mu = \mathcal{O}(1)$  TeV.

Now let us consider an origin of soft  $Z_2$ -parity breaking terms,  $\rho H_u H_{\nu'}$  and  $\rho' H_{\nu} H_d$  in Eq.(2). They play a crucial role of generating the marvelous relation in Eq.(1) as well as a tiny Dirac neutrino mass. Since the values of  $\rho$ ,  $\rho'$  are of order  $\mathcal{O}(1)$  TeV, they might be induced from the SUSY breaking effects. We can consider some possibilities for this mechanism. One example is to introduce a singlet S with odd  $Z_2$ -parity. The superpotential including S below the GUT scale is given by

$$\mathcal{W}_{S}^{eff} = \mu H_{u} H_{d} + M H_{\nu} H_{\nu'} + M' T \bar{T} + M'' T_{\nu} \bar{T}_{\nu} + \mu_{S} S^{2} + \frac{1}{\Lambda} S^{4} - S H_{u} H_{\nu'} - S H_{\nu} H_{d} - S T \bar{T}_{\nu} - S T_{\nu} \bar{T}.$$
 (15)

Denoting  $\langle T \rangle = t$ ,  $\langle \bar{T} \rangle = \bar{t}$ ,  $\langle T_{\nu} \rangle = t_{\nu}$ ,  $\langle \bar{T}_{\nu} \rangle = \bar{t}_{\nu}$ , and  $\langle S \rangle = s$ , the effective potential of the Higgs sector is given by

$$V_{S}^{eff} = |Mv_{\nu} - sv_{u}|^{2} + |Mv_{\nu'} - sv_{d}|^{2} + |M''t_{\nu} - st|^{2} + |M''\bar{t}_{\nu} - s\bar{t}|^{2}$$

$$+ |\mu v_{d} - sv_{\nu'}|^{2} + |\mu v_{u} - sv_{\nu}|^{2} + |M't - st_{\nu}|^{2} + |M'\bar{t} - s\bar{t}_{\nu}|^{2}$$

$$+ |2\mu_{S}s + 4s^{3}/\Lambda - v_{u}v_{\nu'} - v_{\nu}v_{d} - t\bar{t} - t_{\nu}\bar{t}_{\nu}|^{2}$$

$$- m_{S}^{2}s^{2} - m_{H_{u}}^{2}v_{u}^{2} + m_{H_{d}}^{2}v_{d}^{2} + m_{H_{\nu}}^{2}v_{\nu}^{2} + m_{H_{\nu'}}^{2}v_{\nu'}^{2} + m_{T}^{2}t^{2} + m_{T}^{2}\bar{t}^{2} + m_{T_{\nu}}^{2}\bar{t}_{\nu}^{2} + m_{T_{\nu}}^{2}\bar{t}_{\nu}^{2} + \cdots ,$$

$$(16)$$

where we omit D- and B-terms for simplicity. The last line in Eq.(16) are soft SUSY breaking mass squared terms. Taking a parameter region of  $-m_S'^2 \equiv -m_S^2 + 4\mu_S^2 < 0$ , we obtain  $s \simeq \sqrt{\Lambda/(32\mu_S)} \ m_S'$ . Then, when  $m_S' \sim \mu_S \sim 1$  TeV and  $\Lambda \sim 30$  TeV, the suitable  $\rho$ - and  $\rho'$ -terms in Eq.(2) are effectively induced through  $s \sim 1$  TeV. This vacuum also suggests the suitable VEVs of  $v_{u,d} \sim 100$  GeV,  $v_{\nu,\nu'} \sim 0.1$  eV as well as  $t = \bar{t} = t_{\nu} = \bar{t}_{\nu} = 0$ . Unfortunately, the scale of  $\Lambda \sim 30$  TeV is a little artificial in this example. But, this is around  $M_{NP}$ , and much better than inputting  $\mathcal{O}(10)$  eV as the  $Z_2$ -parity breaking soft mass parameters. Another example is to take a non-canonical Kähler of  $[S^{\dagger}(H_uH_{\nu'} + H_{\nu}H_d) + \text{h.c.}]_D$ . Where F-term of S could induce the  $\rho$ - and  $\rho'$ -terms effectively through the SUSY breaking scale as in Giudice-Masiero mechanism[22]. There might be other models which induce the  $\rho$ - and  $\rho'$ -terms in Eq.(2) except for introducing a singlet S.

### 4 Summary

Among three typical energy scales, a neutrino mass scale, a GUT scale, and a TeV (SUSY)-scale, there is a marvelous relation of Eq.(1). In this paper, we have suggested a simple supersymmetric neutrinophilic Higgs doublet model, which realizes the relation of Eq.(1) dynamically as well as the suitable  $m_{\nu}$  through a tiny VEV of neutrinophilic Higgs without additional scales other than  $M_{NP}$  and  $M_{GUT}$ . Usually, SUSY neutrinophilic doublet models have tiny mass scale of soft  $Z_2$ -symmetry breaking as  $\rho, \rho' = \mathcal{O}(10)$  eV. This additional tiny mass scale plays a crucial role of generating the tiny neutrino mass, however, its origin is just an assumption. In other words, the smallness of  $m_{\nu}$  is just replaced by that of  $Z_2$ -symmetry breaking mass parameters, and this is not an essential explanation of tiny  $m_{\nu}$ . This is a common serious problem exists in neutrinophilic Higgs doublet models in general. Our model have solved this problem, where two scales of  $M_{GUT}$  and  $M_{NP}$  naturally induce the suitable magnitude of  $m_{\nu}$  through the relation of Eq.(1), and does not require any additional scales. A gauge coupling unification is also preserved automatically in our setup. We have also considered the embedding in SU(5) GUT and some candidates of inducing the  $Z_2$ -symmetry breaking terms from the SUSY breaking effects.

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